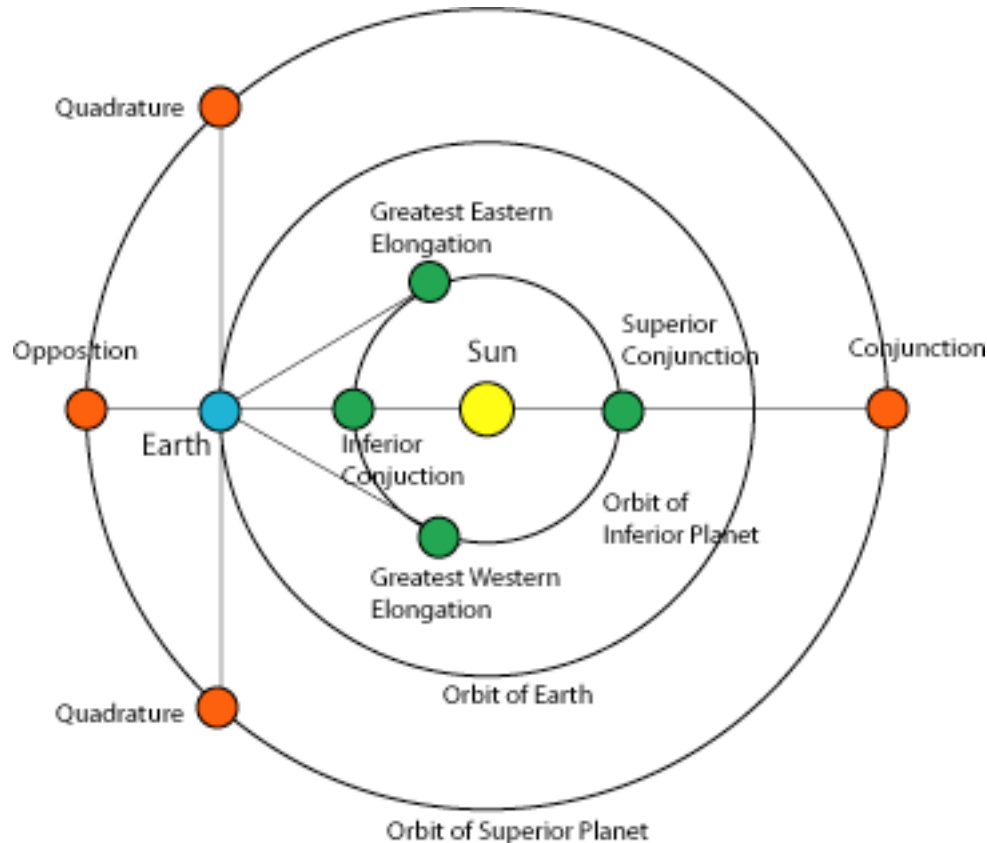


Planetary Configurations

The astronomers of antiquity recognized two types of planets; the inferior planets (Mercury and Venus) were always found in the same part of the sky as the Sun whereas the superior planets (Mars, Jupiter, and Saturn) could sometimes be found in the opposite half of the sky from the Sun. However, both types of planet were always found near the ecliptic, the Sun's annual path around the celestial sphere. A planetary configuration just describes the location on the sky of a planet with respect to the Sun at any particular time. Also, the elongation of a planet is the angle between the planet and the Sun on the sky at any time. The maximum elongation angle for Mercury is about 28° , that for Venus about 48° but the superior planets can have eastern or western elongation angles up to 180° east or west of the Sun. The following figure provides the names for some special planetary configurations.



The apparent motion of a planet depends upon where it is in its sequence of configurations. A planet goes through its full sequence of its configurations and motions (and brightness variations) in a time interval called its Synodic Period. In the above figure all motions are counterclockwise, corresponding an “eastward” sense of motion on the sky. However, the Earth is also moving, which gives rise to the phenomenon of periodic apparent retrograde (westward) motion of a planet on the sky as seen from the Earth. A superior planet (today including Uranus, Neptune, Pluto, Eris, ...) will be observed to undergo retrograde motion when it is being overtaken by the Earth. This is when the planet lies in the shorter arc centered on the Opposition point between Eastern Quadrature and Western Quadrature. An inferior planet appears to be undergoing retrograde motion as it overtakes the Earth and moves from Greatest (or Maximum) Eastern Elongation, through Inferior Conjunction, to Greatest Western Elongation. The duration of apparent retrograde motion is shorter in time that that of direct (eastward) motion on the sky.

Note that when Mars is at quadrature, a Martian would observe the Earth to be at greatest elongation. If we observe Venus at greatest elongation a Venerian would observe the Earth to be at quadrature, etc.

Sidereal and Synodic Periods

The Sidereal Period of an object is the time it takes to make a complete circuit of the celestial sphere with respect to the fixed stars. The Synodic Period is the time required to make a circuit of the celestial sphere with respect to the Sun - which is also moving eastward among the stars. The synodic period is always greater than the sidereal period, the difference being due to the apparent motion of the Sun. The sidereal period of the Moon is the time it takes to go once around the Earth; the sidereal period of a planet is the time it takes to go once around the Sun. *i.e.*, its orbital period.

Consider two objects (*e.g.*, planets) orbiting the Sun. For simplicity, we will assume that their orbits are circular and that they move in the same direction and at constant speeds along their respective paths. Each object has an orbital period, P , which is the time required to make one circuit of its path so that after some period of time T an object will have completed $N = T/P$ circuits of its orbit. The two objects will, in general, have different orbital periods. If they are initially lined up with the Sun at some time (so that the inner sees the other to be at opposition, say) then the planet with the shorter orbital period will pull ahead of the other but eventually overtake it, having completed one more orbit than the slower object, and the initial alignment will be restored. The period of time required for this to happen is called the synodic period. How long is that? The requirement is that in one synodic period the planet with the shorter orbital period will have completed one (1) more orbit than the planet with the longer period. Let the two orbital periods be P_{longer} and P_{shorter} and P_{synodic} be the synodic period for this planet-pair. Our condition is

$$(P_{\text{synodic}}/P_{\text{shorter}}) = (P_{\text{synodic}}/P_{\text{longer}}) + 1$$

- namely that the object with the shorter period has completed one more orbit than the other planet. This equation can be rewritten in a more convenient form as

$$1/P_{\text{synodic}} = 1/P_{\text{shorter}} - 1/P_{\text{longer}}$$

As it happens, outer bodies in our solar system (or in any other system) have longer orbital periods than inner bodies (*cf.* Kepler's Third Law) so we could rewrite this as

$$1/P_{\text{synodic}} = 1/P_{\text{inner}} - 1/P_{\text{outer}}$$

Some Examples:

* Mars is observed to have a synodic period of 2.135 years and we know the Earth (which is the inner planet in this case) has an orbital period of 1 year, by definition. Inserting these numbers in the above expression, and solving for the Martian orbital period we find that Mars completes an orbit about the Sun in 1.881 years

* Venus has an orbital (sidereal) period of 0.615 years. The synodic period as seen from the Earth (which is now the outer planet with orbital period one year) is 1.599 years.

* A Martian observing Venus (or *vice versa*) would observe it to have a synodic period (in years) given by

$$1/P_{\text{synodic}} = 1/0.615 - 1/1.881$$

or $P_{\text{synodic}} = 0.914$ years. (0.914 Earth years corresponds to 0.49 Martian years or 1.49 Venerian years.)

* The Moon makes a circuit of the celestial sphere in the sidereal month of 27.32 days. The Sun does so in the sidereal year of 365.26 days. The synodic month (the month of the phases) in days is therefore

$$1/P_{\text{synodic}} = 1/27.32 - 1/365.26 \quad \text{or} \quad P_{\text{synodic}} = 29.52 \text{ days}$$

Note that planetary orbits (and that of the Moon) are elliptical rather than circular and that the speed of planets in their orbits are not quite constant. As a result the actual time intervals between successive oppositions, say, can vary slightly from the synodic period.

Another Example:

The hour hand on a clock makes one circuit of the dial in 12 hours, the minute hand does so in 1 hour. The two line up at 12:00. At what time will they line up next? Answer: At about 1:05:27

$$1/P_{\text{synodic}} = 1/1 \text{ hour} - 1/12 \text{ hours} \quad \text{or} \quad P_{\text{synodic}} = 12/11 \text{ hours} = 1^{\text{h}} 27^{\text{m}} 27.27^{\text{s}}$$